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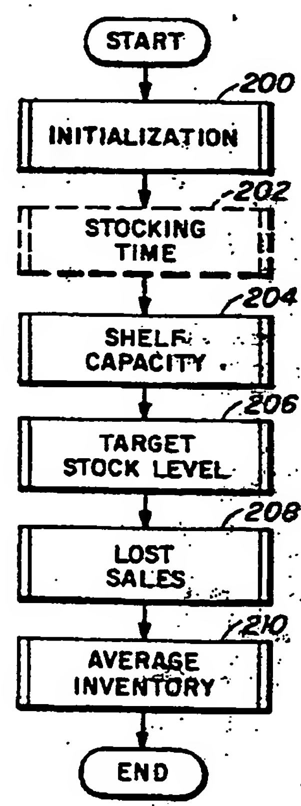
(54) Title: RETAIL SHELF INVENTORY SYSTEM

(57) Abstract

A retail shelf inventory management system includes memory for storing inventory data. The inventory data includes a study period and a retail opening time and a retail closing time for each day within the study period and historical inventory data for the study period for each of a number of different inventory items or products. The historical inventory data includes a movement value, a customer service level, a proportion of business done for each day within the study period, and a variability of demand. An optimizing program and processing unit are operatively connected to the memory devices for optimizing a shelf capacity (204) for any or each of said plurality of inventory items. Both user selected shelf capacity values and the optimized shelf capacity value can be separately utilized for calculating target stock levels (205), order-to-levels, estimated sales estimated lost sales (208), average inventory values (210) and customer service levels achieved.



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RETAIL SHELF INVENTORY SYSTEM

BACKGROUND OF THE INVENTION

Field of the Invention

The present invention relates generally to inventory management systems and more particularly to data processing methodology and system for retail shelf inventory management.

Description of the Prior Art

Computerized inventory management systems are known. A publication entitled "INFOREM Principles of Inventory Management Application Description," second edition May, 1978 copyright IBM Corporation, provides a description of general principles of inventory management and how these principles are implemented in the INFOREM application program for inventory management developed by IBM Corporation. However, such computerized inventory systems have not adequately addressed the problem of providing a retail shelf inventory management system having data processing methodology capable of identifying an optimum shelf capacity on an individual product basis and utilizing the identified optimum shelf capacity for generating reordering time schedules and quantities for individual products.

Summary of the Invention

A principal object of the present invention is to provide an improved retail shelf inventory management system. Other important objects of the present invention are

to provide an improved retail shelf inventory management system capable of identifying an optimum shelf capacity on an individual product basis; to provide such a system capable of identifying target stock levels and order-to-levels separately utilizing the identified optimum shelf capacity and/or a user supplied shelf capacity value; and to provide such a system capable of identifying estimated sales and customer service levels achieved.

In brief, the objects and advantages of the present invention are achieved by a retail shelf inventory management system including memory for storing inventory data. The inventory data includes a study period and a retail opening time and a retail closing time for each day within the study period and historical inventory data for the study period for each of a number of different inventory items or products. The historical inventory data includes a movement value, a customer service level, a proportion of business done for each day within the study period, and a variability of demand. Optimizing means are operatively connected to said memory means for optimizing a shelf capacity for any or each of said plurality of inventory items. Both user selected shelf capacity values and the optimized shelf capacity value can be separately utilized for calculating target stock levels, order-to-levels, estimated sales, estimated lost sales, average inventory values and customer service levels achieved.

Brief Description of the Drawings

The present invention together with the above and other objects and advantages may best be understood from the following detailed description of the embodiment of the invention illustrated in the drawings, wherein:

FIG. 1 is a block diagram representation of a retail shelf inventory system according to the present invention; and

FIGS. 2-7 are flow charts illustrating the logical steps performed by the retail shelf inventory system of FIG. 1.

Detailed Description of the Preferred Embodiment

Referring now to the drawings, in FIG. 1 there is illustrated a block diagram representation of a retail shelf inventory system generally designated by the reference character 10.

Appendix I below sets forth a detailed description of a stochastic inventory model according to the present invention. Appendix I provides a problem definition and provides the results by the stochastic inventory model of the invention and includes both a mathematical description and a corresponding exemplary program specification written in Pascal for implementing the mathematical description.

As illustrated in FIG. 1, the retail shelf inventory system 10 includes a central processing unit 12 and an associated memory generally designated by the reference character 14. As shown, the memory 14 includes a program memory space 16 for storing the retail shelf inventory program of the invention and a plurality of data memory spaces 18 for storing multiple data files (1)-(Q) including both user supplied data and computed results data of data sets and arrays. A keyboard 20 is coupled to the central processing unit 12 for entering user selections and data. A display 22 and a printer 24 are also coupled to the central processing unit 12 for reporting calculated results to the user. The retail shelf inventory system 10 can be implemented by a personal computer system, for example, such as, an IBM PS/2 Model 80 with an associated disk drive 70M bytes memory, or various other commercially available microcomputer based systems.

As set forth below in Appendix I under heading "B. Results from the model", the retail shelf inventory system 10 provides an optimum shelf capacity on an individual product basis. Optimum stocking times when a shelf

should be replenished also can be provided by the retail shelf inventory system 10. For both the calculated ptimum shelf capacity and each of the user entered shelf capacity values, the inventory system 10 generated results can include order-to-quantities for reordering, estimated lost sales and average inventory stock values for both the store and shelf stock as seen by a customer. In accordance with a feature of the invention, the daily opening and closing store times and historical values of the proportions of business done by day within a study period are supplied by the user and utilized for generating the user selected results to be reported for a particular product. The use of actual opening and closing store times together with the proportional business done values simplifies mathematical calculations and can provide increased accuracy for generated results.

Referring now to FIG. 2, there is shown a flow chart generally illustrating the overall program logic sequence in accordance with the principles of the invention for the shelf inventory system 10. The program starts with an INITIALIZATION routine performed indicated at a block 200. The INITIALIZATION routine is illustrated and described with respect to FIG. 3 and in Appendix I under the heading "Solution: Mathematical Description", subheadings "1. Notation" and "2. Preliminaries"; under the heading "Solution: Program Specifications," subheadings "1. Variables" and "2. Initialisation."

Then an optional STOCKING TIME routine indicated at a block 202 indicated at a block 204 shown in dotted line can be performed to provide optimum stocking times when a shelf should be replenished. At least an initial stocking time is supplied by the user, then subsequent optimal stocking times are calculated to space the subsequent stockings evenly in shop time over the study period. Subsequent optimal stock times are identified by calculating an inverse function of clock as defined in Appendix I

under the heading "Solution: Mathematical Description," subheading "3. Optimal stocking times."

Next an optimum SHELF CAPACITY routine indicated at a block 204 is performed for providing an optimum shelf capacity value. The optimum SHELF CAPACITY routine is illustrated and described with respect to FIG. 4 and in Appendix I under the heading "Solution: Mathematical Description," subheading "4. Calculating optimal shelf capacity" and under the heading "Solution: Program Specifications," subheading "3. Calculating shelf capacity."

Next a TARGET STOCK routine indicated at a block 206 is performed for calculating target stock levels and order-to-levels. The TARGET STOCK routine is illustrated and described with respect to FIG. 5 and in Appendix I under the heading "Solution: Mathematical Description," subheading "5. Calculating target stock levels, and order-to levels" and under the heading "Solution: Program Specifications," subheading "4. Calculating target stock levels, and order-to levels."

Next a LOST SALES routine indicated at a block 208 is performed for calculating the expected lost sales and the customer service level achieved for the study period. The LOST SALES routine is illustrated and described with respect to FIG. 6 and in Appendix I under the heading "Solution: Mathematical Description," subheading "6. Lost sales" and under the heading "Solution: Program Specifications," subheading "5. Lost sales."

Next an AVERAGE INVENTORY routine indicated at a block 210 is performed for calculating the expected average shelf inventory seen by the customer, the expected average store inventory at the close of business on a particular day within the study period and the expected total store amount. The AVERAGE INVENTORY routine is illustrated and described with respect to FIG. 7 and in Appendix I under the heading "Solution: Mathematical Description," subheading "7. Average inventory calculations" and under the

heading "Solution: Program Specifications," subheading "6. Average inventory calculations."

Referring to FIG. 3, there is shown a flow chart illustrating the sequential steps performed during the INITILIZATION routine. The INITILIZATION routine begins with the definition of initial and default values for constants, variables, and functions utilizing user supplied data structure definitions. The user supplied data is set forth in Appendix I under the heading "A. factors influencing the model." The user supplied data is included within the list of variables used in the program in Appendix I under the heading "Solution: Program Specifications, subheading "1. Variables" as defined under the heading "Solution : Mathematical Description", subheadings "1. Notation" and "2. Preliminaries." The user supplied data is read and copied into the defined data structures, then precalculations are performed on the user supplied data on an individual product basis indicated at a block 300.

As illustrated in Appendix I under the heading "Solution: Program Specifications, subheading "2. Initialisation," the user supplied time schedule values are converted from real time into shop time and the proportion of total business done and the modified mean demand in each interstocking period is calculated indicated at a block 302 and described with reference to FIG. 3 as follows.

First an order shop time array and a delivery shop time array is calculated utilizing the user supplied store opening and closing time for each day J=0, ..., L-1 in the study period and user supplied order and delivery real time schedule indicated at a block 304. Next when stocking times are user supplied, the user supplied stocking times are converted to shop time to generate a stocking shop time array indicated at a block 306. Otherwise when only an initial stocking time is user supplied, then the stocking shop time array is calculated indicated at the block 306. Next the proportion of business done and the modified mean demand during each stocking interval is

calculated indicated at a block 308. Error checking routines indicated at blocks 310, 312 are sequentially performed on the calculated results for each of the above described steps. When errors are not detected at the block 310, the results are stored indicated at a block 314 in predefined corresponding data structure for results of supply, stock and demand data sets.

Referring now to Fig. 4, there is shown a flow chart illustrating the logical steps of the SHELF CAPACITY routine performed for calculating an optimal shelf capacity. Appendix I under the heading "Solution: Mathematical Description", subheading "4. Calculating optimal shelf capacity" and under the heading "Solution: Program Specifications, subheading "3. Calculating shelf capacity" describes the SHELF CAPACITY routine in detail. The sequential steps begin by identifying one of Methods 1 indicated at a block 400, Method 2 indicated at a block 402 or Method 3 indicated at a block 404 to be utilized for calculating the optimal shelf capacity. A particular Method 1, 2, or 3 is performed responsive to a user entered method selection for the optimum shelf capacity calculation. Next for either of the Methods 1, 2, and 3, historical demand inventory data for the particular product, the proportion of business done, the modified mean demand during each interstocking interval and the desired customer service level is collected indicated respectively at block 406, 408 or 410.

When Method 1 is selected a root finding routine is performed to identify the optimum shelf capacity C. This root finding routine evaluates repeatedly the expected overall lost sales indicated at a block 412 for the entire study period utilizing user supplied data, the calculated values of proportions of business done and modified mean demand for each interstocking period, the demand and the customer service level to be achieved as set forth in Appendix I under the subheadings "Method 1." If the root finding fails indicated at a block 414, then an error flag is set and no value is stored. Otherwise the result from

the root finding calculation is stored indicated at a block 416 as the optimum shelf capacity.

When Method 2 is utilized for finding the optimum shelf capacity C, then an index k is found for which the proportion of business done in the j^{th} interstocking period is greatest. Then the optimum shelf capacity is identified to achieve the desired customer service level during the identified busiest interstocking period indicated at a block 418 rather than the entire study period utilized in Method 1. The Method 2 routine is set forth in Appendix I under the subheadings "Method 2." When an error indicated at a block 420 does not result from the calculation of the optimum shelf capacity of Method 2, then the computed optimum shelf value is stored indicated at a block 422.

Method 3 identifies an optimum value for shelf capacity utilizing a block of the busiest or worst r consecutive days identified in the entire study period indicated at a block 424 as set forth in Appendix I under the subheadings "Method 3." The worst r consecutive days can occur within a single or multiple interstocking periods. When an error indicated at a block 426 does not result from the calculation of the optimum shelf capacity of Method 3, then the computed optimum shelf value is stored indicated at a block 428.

When the user has supplied one or more values for the shelf capacity indicated at a block 430, for example, such as, a current shelf capacity and a selected shelf capacity then the user defined C value or values are stored indicated at a block 432 for subsequent computations.

Referring to FIG. 5, there is shown a flow chart illustrating the TARGET STOCK LEVEL routine. The sequential steps begin with identifying first a stored shelf value C indicated at a block 500. The identified stored shelf value C can be either the calculated optimum shelf value or one supplied by the user. Next the expected sales in the j^{th} interstocking period are calculat d indicated at a block 502 using the formula set forth in Appendix I under

the heading "Solution: A Mathematical Description", subheading "Method 3." Then an effective delivery time is calculated indicated at a block 504 as set forth in paragraph (5i) in Appendix I under the heading "Solution: Mathematical Description", subheading "5. Calculating target stock levels, and order-to levels." Next an internal target stock level value W_k is calculated indicated at a block 506 using the formula set forth under paragraph (5). The calculated internal stock level is then stored for subsequent computations. Next an order-to-level is calculated indicated at blocks 508, 510 utilizing the shelf value C and the stored internal target stock level by calculating the expected sales between the order time and the delivery time using the formulas set forth under paragraphs (5a),(5b). The order-to-level is identified to ensure that the stored internal target stock level value W_k is satisfied at the delivery time.

Referring now to FIG. 6, there is shown a flow chart illustrating the logical steps performed for the LOST SALES routine. The LOST SALES routine is described in Appendix I under the heading "Solution: Mathematical Description," subheading "6. Lost sales" and under the heading "Solution: Program Specifications," subheading "5. Lost sales." The sequential steps begin with a calculation indicated at a block 600 of the effective stock level that is recursively calculated starting from a selected index k for which internal calculated target stock level is maximal utilizing the formulas set forth in paragraphs (6), (6a), and (6b). The calculated effective stock levels are then stored for subsequent computations. Then the expected lost sales for the whole study period is calculated indicated at a block 602 in accordance with the formula set forth in paragraph 6(c). Then the customer service level CSL achieved is calculated indicated at a block 604 using the resultant expected lost sales value utilizing the formula set forth in paragraph 6(d).

Referring now to FIG. 7, there is shown a flow chart illustrating the sequential steps of the AVERAGE INVENTORY routine. Appendix I under the heading "Solution: Mathematical Description", subheading "7. Average inventory calculations" and under the heading "Solution: Program Specifications, subheading "6. Average inventory calculations" describes the AVERAGE INVENTORY routine in detail. The sequential steps begin with calculations indicated at a block 700 of the expected amount of backroom stock b and shelf stock a just after the j^{th} stocking time, utilizing a computed expected sales value utilizing the formulas of paragraphs 7(a) and 7(b). Next the functions a(t) and b(t) are interpolated indicated at a block 702 between the calculated a, b values to identify average stock amount on the shelf and in the backroom in accordance with the formulas of paragraphs 7(c) and 7(d). Next numerical integration of the a(t) time value is calculated indicated at a block 704 as set forth under paragraph (i) "Average Amounts Seen by the Customer." Next the average store inventory is calculated indicated at a block 706 utilizing store time as set forth under paragraph (ii). Then a numerical integration routine can be performed indicated at a block 708 to provide a real time average store inventory as set forth under paragraph (iii).

Then the sequential functions or selected functions of the TARGET STOCK LEVEL, LOST SALES and AVERAGE INVENTORY routines are repeated for the next stored shelf capacity value C.

APPENDIX I

STOCHASTIC INVENTORY MODEL

Problem Definition

To derive, on an individual product basis, a stochastic inventory model using the factors, given below, influencing sales and hence the stock level and re-order strategies.

A. Factors influencing the model

1. The study period; variable from 1 day to a year.
2. The daily opening times; for each day in the study period, the opening and closing times of the store are stated.
3. The demand cycle; this is derived from the following historical data:
   (i) movement figure over the period
   (ii) customer service level over the period
   (iii) distribution of business by day (i.e. sales percentages by day)
   (iv) variability of demand over the period.
4. The number of times the shelf is replenished during the study period.
5. The stocking schedule; each time and day within the study period when the shelf is replenished is stated.
6. The number of deliveries to the store during the study period.
7. The order and delivery schedule; each time and day when an order is raised, i.e. the time at which the amount to be re-ordered is calculated, together with its associated time and day of delivery within the study period.
8. The target stock levels; the total amount of stock in the store after each delivery.
9. The actual shelf capacit; , the amount of the product currently displayed on the shelf.
10. The desired customer service level.

[*User definition of factors*

Obligatory

A1 to A4

First stocking time in A5

A6 to A7, A10.

Optional

Rest of stocking schedule in A5

A8 and A9.

(If A5 and A8 are not fully defined by the user, then the optimal values are produced and used by the model.)]

B. Results from the model

1. Optimal stocking times, the times and days during the study period when the shelf should be replenished.

   [Only used if A5 not fully defined by the user.]

2. Optimal shelf capacity; the amount of the product which should be displayed on the shelf to meet the factors A1 to A5.

3. For each of the shelf capacities (i.e. both Actual and Optimal) the following results may be derived:

   (i) Re-order schedule; for each order point a stock level which the store must re-order to (hence any stock in the store and any stock currently on order must be taken into account when calculating the amount to re-order).

   (ii) Lost sales; the sales not achieved by accepting this shelf size and accounting for the backroom.

   (iii) Average inventory; the average amount of stock in the store.

   (iv) Average amount on shelf; the average amount of stock on the shelf as seen by the customer.

4. Actual customer service level; the customer service level actually achieved when using the actual shelf capacity (A9) rather than the optimal shelf capacity (B2).

Solution: Mathematical Description

1. Notation

L	length of study period (in days)
s_j	time of opening on day j, $j = 0, ..., L-1$
t_j	time of closing on day j, $j = 0, ..., L-1$
μ'	historical movement figure for the study period
$100\beta\%$	desired customer service level
$100\alpha\%$	historical customer service level
θ_j	proportion of business done on day j
$100v\%$	variability of demand
N	number of stockings of shelf
T_k	time of k^{th} stocking of shelf, $k = 0, ..., N-1$
M	number of deliveries
D'_j	time of j^{th} delivery, $j = 0, ..., M-1$
O_j	time of j^{th} order, $j = 0, ..., M-1$
W_j	j^{th} target stock level, $j = 0, ..., M-1$
C^*	actual shelf capacity

2. Preliminaries

(i) We input the historical movement figure μ' and the historical customer service level α, but what *really* matters is the *demand*, μ, which is the number of times someone wants to buy one of the product during the study period. Of course, sometimes a customer who wants the product will find the shelf empty and be unable to get the item, but this will only happen in a proportion $(1-\alpha)$ of cases. Otherwise, the customer will take the item from the shelf, and the movement figure goes up by one. Hence

$$\mu' \equiv \text{movement} = \text{demand} \times \text{historical customer service level}$$

$$= \mu\alpha,$$

so demand $\mu = \mu'/\alpha$.

(ii) If the variability of demand is v, the standard deviation will be μv, and so the *variance of demand*, σ^2, will be $(\mu v)^2$.

(iii) To calculate the proportions of business done in each interstocking period, we make use of the function clock: $[0,L] \rightarrow [0,1]$, which is defined as

$$\text{clock}(t) \equiv \text{proportion of total business done by time } t$$

$$= \sum_{j=0}^{L-1} \theta_j \left\{ \frac{(t-s_j)^+}{t_j - s_j} \wedge 1 \right\} .$$

We subsequently extend clock periodically to the whole of the real line, so that

$$\text{clock}(nL + t) = n + \text{clock}(t)$$

for integers n, and t in [0,L]. The purpose of clock is to convert the real time t into the *shop time* clock(t). As far as the shop is concerned, it is shop time which matters; business proceeds at a constant rate in shop time.

(iv) We also assume that the study period repeats itself, so that if $(T_0,T_1]$ is the zeroth interstocking period, the $(N-1)^{th}$ is $(T_{N-1}, L + T_0]$, for example. The proportion of business done in the j^{th} interstocking period is simply

$$p_j = \text{clock}(T_{j+1}) - \text{clock}(T_j) ,$$

where $T_N = L + T_0$.

(v) We model the demand in the j^{th} interstocking period by the random variable Y_j, which is the positive part of a normal random variable X_j, whose variance is $p_j\sigma^2$ and whose mean μ_j is chosen so that the expected value of Y_j, EY_j, is equal to the mean demand in the period, μp_j. More explicitly, if we define the function A by

(1) $$A(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} - z\,\overline{\Phi}(z) ,$$

where $\overline{\Phi}(z) = P(Z > z)$ for Z a $N(0,1)$ random variable, then

(2) $$A(z) = E(Z-z)^+ ;$$

thus we choose μ_j to solve

(3) $$\mu p_j = \sigma\sqrt{p_j}\, A\left(\frac{-\mu_j}{\sigma\sqrt{p_j}}\right).$$

[Note: The choice of a normal distribution for Z is essentially arbitrary, and *any* zero-mean unit-variance distribution could serve instead. The expression (2) for A stays the same, but the explicit formula (1) will change.]

3. Optimal stocking times

Suppose we have been told the time T_0 of the zeroth stocking, which occurs at the shop time $\tau_0 = \text{clock}(T_0)$. Then the best way to choose subsequent stocking times is to space them out evenly through the study period - but 'evenly' in terms of shop time. Thus we define

$$\tau_j = \tau_0 + j/N, \qquad j = 1, ..., N-1,$$

(with addition mod 1) and set the recommended stocking times $T_1, ..., T_{N-1}$ to be

$$T_j = \text{clockinv}(\tau_j),$$

where clockinv is the inverse function to clock; $\text{clockinv}(t) = \inf\{u: \text{clock}(u) > t\}$. This function can be expressed explicitly in terms of the θ_j, s_j, t_j and L very much as clock can. Less satisfactorily, it can be evaluated by a general root-finding routine from clock.

The user can, of course, ignore the recommended T_j and supply his own.

4. Calculating optimal shelf capacity

If the shelf capacity is C, then assuming that the shelf is full at the beginning of the j^{th} interstocking period, the expected lost sales in the j^{th} interstocking period is given by the formula

$$(4) \qquad \lambda_j(C) = \sigma\sqrt{p_j}\, A\left(\frac{C-\mu_j}{\sigma\sqrt{p_j}}\right),$$

which is $E(Y_j - C)^+$.

There are now three possible ways of choosing the optimal shelf capacity, depending on the user's performance criteria. We consider the first to be the most natural. As usual, the user is free to ignore the recommended shelf capacity, and input his own value C^* instead.

Method 1. This method should be used if the aim is to ensure that on average $100\beta\%$ of *the total demand throughout the study period* will be met. Assuming that there is always enough available at the start of an interstocking period to fill the shelf, the expected lost sales in the study period is

$$l(C) = \sum_{j=0}^{N-1} \lambda_j(C)$$

when the shelf capacity is C, and we simply pick C so that

$$l(C) = (1-\beta)\mu.$$

This will need a root-finding routine.

Method 2. This method will ensure that *during each interstocking period*, at least $100\beta\%$ of the demand will be met. Find the index k for which p_j is greatest, and then choose C to satisfy

$$\lambda_k(C) = (1-\beta)\mu p_k .$$

This choice of C will always be higher than in Method 1.

Method 3. This method should be used if the aim is to have the shelf large enough to carry through *the worst r consecutive days* in the study period. To do this, let p be the maximum of $\theta_j + \cdots + \theta_{j+r-1}$ as j varies, calculate ν as in (3):

$$\mu p = \sigma\sqrt{p}\, A\left(\frac{-\nu}{\sigma\sqrt{p}}\right),$$

and now choose C as in Method 2:

$$\sigma\sqrt{p}\, A\left(\frac{C-\nu}{\sigma\sqrt{p}}\right) = (1-\beta)\,\mu p .$$

At this stage, the program has a value of C to work with, either computed by one of the three methods, or else input by the user. Now one can compute, for example, the expected sales in the j^{th} interstocking period:

$$S_j = S_j(C) = E(Y_j \wedge C) = \mu p_j - \lambda_j(C) .$$

5. Calculating target stock levels, and order-to levels

The user inputs the number M of deliveries in the study period, the times $D'_0, \ldots, D'_{M-1}$ at which they are made, and the times $O_0, \ldots, O_{M-1}$ at which the corresponding orders are placed. Some retailers may require to be protected against delays in delivery of up to δ, in which case we replace D'_j by $D'_j + \delta$, and since we cannot make use of delivered goods until the next stocking after delivery, we immediately replace D'_j by the *effective* delivery time

(5i) $D_j = \inf\{T_k : T_k > D'_j\}$.

N w if $D_k = T_r$, $D_{k+1} = T_{r+n+1}$ $(n \geq 0)$, our *target stock level* W_k at the delivery time D_k will be set so as to meet on average the sales to be made in $(T_r, T_{r+n+1}]$:

(5) $$W_k = \left(\sum_{j=r}^{r+n} E(Y_j \wedge C) \right) \vee C \ .$$

We take the maximum with C so as to ensure that we start with a full shelf at D_k. The objective is to ensure that just after the k^{th} delivery has been made, the total amount in the shop (shelf + backroom) is W_k (or more). Thus we choose the *order-to level* V_k to achieve this, as follows.

Firstly we calculate (an upper bound for) the expected sales in $(O_k,D_k]$, by summing the expected sales S_j for each interstocking interval j contained entirely in $(O_k,D_k]$, and then adding a correction for the sales between O_k and the next stocking time. Explicitly, if O_k occurs in interstocking period m, shop time t after the beginning of the interstocking period, then the expected sales between O_k and the end of interstocking period m is

5(a) $$S_m - E\left[\left(\frac{tY_m}{p_m}\right) \wedge C\right] = S_m - \frac{tEY_m}{p_m} + E\left(\frac{tY_m}{p_m} - C\right)^+ ;$$

this is the correction we add in, to arrive at the expected sales U_k during $(O_k,D_k]$. We now define the k^{th} order-to level by

5(b) $$V_k = W_k + U_k \ .$$

The interpretation is this. At time O_k, we have amount x in the shop (shelf + backroom) and we have placed orders for a total amount y which has not yet been delivered; we therefore place an order for amount $(V_k - x - y)^+$.

6. Lost sales

We have chosen a shelf capacity C, and target stock levels $W_0, ..., W_{M-1}$. Suppose that $D_k = T_r$, $D_{k+1} = T_{r+n+1}$. Then the lost sales in $(D_k,D_{k+1}]$ is exactly

$$\sum_{j=r}^{r+n} Y_j - \left(W_k \wedge \sum_{j=r}^{r+n} (Y_j \wedge C) \right).$$

The expected value of this is too cumbersome to evaluate explicitly, so we approximate the expected lost sales in $(D_k,D_{k+1}]$ by

(6) $$\Lambda_k(C,W_k) \equiv \mu \sum_{j=r}^{r+n} p_j - \left(W_k \wedge \sum_{j=r}^{r+n} E(Y_j \wedge C) \right).$$

We could now just sum the $\Lambda_k(C,W_k)$ to yield expected lost sales, but this would be erroneous for the following reason. Suppose that $W_1, ..., W_{M-1}$ were all very small, but

W_0 was enormous. Then there would be lots of unsold stock at time D_1, so *effectively* there would be more in the shop at time D_1 than W_1. What we need to do is replace the target stock levels W_k by the *effective* target stock levels $\tilde{W}_k$, and then proceed.

We calculate the effective target stock levels $\tilde{W}_k$ as follows. Pick the index k for which W_k is maximal; suppose for convenience that it is $k = 0$. Then we set

6(a) $$\tilde{W}_0 = W_0 ,$$

and define recursively

6(b) $$\tilde{W}_{i+1} = W_{i+1} \vee \{\tilde{W}_i - \mu(\text{clock}(D_{i+1}) - \text{clock}(D_i)) + \Lambda_i(C,\tilde{W}_i)\} .$$

This is because if we had $\tilde{W}_i$ in the shop at D_i, the demand in $(D_i,D_{i+1}]$ would be $\mu\{\text{clock}(D_{i+1}) - \text{clock}(D_i)\}$, of which on average $\Lambda_i(C,\tilde{W}_i)$ would be lost. The amount remaining just before D_{i+1} would be $\tilde{W}_i - \mu\{\text{clock}(D_{i+1}) - \text{clock}(D_i)\} + \Lambda_i(C,\tilde{W}_i)$, and this might exceed W_{i+1}.

Finally, then, we calculate the expected lost sales for the whole study period:

6(c) $$\text{Expected lost sales} = \sum_{k=0}^{M-1} \Lambda_k(C,\tilde{W}_k) .$$

From this follows the customer service level achieved:

6(d) $$100\,[1 - (\text{Expected lost sales})/\mu]\% .$$

7. Average inventory calculations

The first step here is to calculate the expected amount on the shelf and in the backroom just before and just after the j^{th} stocking time, T_j.

Suppose that just after T_j, there is a_j on the shelf and b_j in the backroom. Just before T_{j+1}, there is b_j in the backroom and (approximately) $(a_j - S_j)^+ = (a_j - E(Y_j \wedge C))^+$ on the shelf. Thus

7(a) $$b_{j+1} = (b_j - (S_j \wedge a_j))^+$$

7(b) $$a_{j+1} = ((a_j - S_j)^+ + b_j) \wedge C$$

except when T_j is a delivery time D_k, in whicvh case $b_j + a_j$ is simply the effective target stock level $\tilde{W}_k$, and a_j is just $\tilde{W}_k \wedge C$.

The second stage is to interpolate between these values to obtain functions $a(\cdot)$, $b(\cdot)$

for the average amount on the shelf and in the backroom. The backroom stock level is easy:

$$7(c) \quad b(t) \equiv b_j \qquad (\tau_j \le t < \tau_{j+1}) .$$

For the amount on the shelf, we set

$$7(d) \quad a(t) = (a_j - \mu t) \vee (a_j - S_j)^{+} \qquad (\tau_j \le t < \tau_{j+1}) .$$

[Recall that $\tau_j \equiv \text{clock}(T_j)$.] From this, we can obtain the average amount on the shelf (or in the backroom), however one may choose to define it. We suggest some possibilities here.

(i) *Average amount seen by the customer.* This is simply

$$\int_0^1 a(t)dt \ ,$$

which can be evaluated by a (trapezium rule) numerical integration.

(ii) *Average amount in the store at the end of the day's trading.* This amount in the shop at the close of trading on day j is simply

$$\rho_j \equiv a(\text{clock}(t_j)) + b(\text{clock}(t_j)),$$

so the average amount at the end of the day's business is just $\left(\sum_{j=0}^{L-1} \rho_j\right) / L$.

(iii) *Average amount in store.* (This is the 'true' average, averaged in real time.) This is calculated simply as

$$\int_0^L \{a(\text{clock}(s)) + b(\text{clock}(s))\} ds / L \ ,$$

again a candidate for the numerical integration routine.

Solution: Program Specifications

There follows a largely complete Pascal program to carry out the calculations laid out in the mathematical description.

1. Variables

```
type
    vec  =  array [0..MAX] of real;
    daily  =  array [0..365] of real;
    vec_int  =  array [0..MAX] of integer;
```

{the parameter MAX should be set large enough to cope with the biggest value of nstock, the number of stockings. MAX = 700 will probably do.}

{There follows a list of the variables used in the program, together with their type, range of possible values, and symbol used in the mathematical description (if applicable). It is assumed that all the variables above the line have already been input.}

Variable name	type	range	math. symbol
nday	integer	1,...,365	L
opentimes	daily	array of increasing reals in [0,1]	(s_j)
closetimes	daily	array of increasing reals in [0,1]	(t_j)
hist_movement	real	positive integer	μ'
CSL	real	(0,1)	β
hist_CSL	real	(0,1)	α
proportions	daily	reals in [0,1]	(θ_j)
variability	real	positive real	v
nstock	integer	0,...,MAX	N
stock_true_time[1],[2]	vec	reals in [0,1]	(T_i)
ndel	integer	0,...,MAX	M
del_true_time[1]	vec	reals in [0,1]	(D'_k)
order_true_time[1]	vec	reals in [0,1]	(O_k)
delay_protect	real	[0,1]	δ
C[3]	real	positive real	C^*
target_stock_levels[4]	positive reals	(W_j)	

---

demand	real	positive real	μ
sigma	real	positive real	σ
stock_shop_time	vec	reals in [0,1]	(τ_k)
del_shop_time	vec	reals in [0,1]	
order_shop_time	vec	reals in [0,1]	
p	vec	reals in [0,1]	(p_j)
mod_demands	vec		(μ_j)
sales	vec	positive reals	(S_j)
raw_tsl	vec	positive reals	
target_stock_levels	vec	positive reals	(W_j)
effective_tsl	vec	positive reals	$(\bar{W}_j)$
shelf	vec	positive reals	(a_j)
backroom	vec	positive reals	(b_j)
order_to	vec	positive reals	(V_k)
var	real	positive real	σ^2
h	real	positive real	
flag	vec_int	-1,0,...,MAX-1	
lead_time_sales	vec	positive real	
lost_sales	real	positive real	
CSL_achieved	real	real in [0,1]	

---

(1) The entries stock_true_time[0],..., stock_true_time [nstock-1] are the successive times at which stockings are made. All subsequent entries in the array are initialised to 0. Similarly for del_true_time, order_true_time.

(2) The program contains a facility for calculating the array stock_true_time from the input value stock_true_time[0] alone; so in some uses, it would be satisfactory to input only this one value.

(3) There may be no value of C offered, because the main program has the capacity to calculate C.

(4) The user may wish to input the target stock levels (one for each of the ndel deliveries), but the program will calculate these if they are not input.

2. Initialisation

```
begin
    epsilon := 1.0E-6;
    variability := max (variability, epsilon);
    demand := hist_movement/hist_CSL;
    sigma := demand*variability;
    var := sigma*sigma;
    h := 1.0/nstock;
    stock_true_time [nstock] := stock_true_time[0];
    del_true_time [ndel] := del_true_time[0];
    order_true_time [ndel] := order_true_time[0];
{These conventions make 'wrap round' easier.}
  for j := 0 to ndel do
    begin
        del_shop_time[j] := clock(del_true_time[j], opentimes, closetimes,
                                  proportions, nday);
        order_shop_time[j] := clock(order_true_time[j], opentimes, closetimes,
                                  proportions, nday);
    end;
{It may be that the user has input only stock_true_time[0], and is leaving it to the program
to calculate the rest of the array. This is done as follows:
  stock_shop_time[0] := clock(stock_true_time[0], opentimes, closetimes,
                              proportions, nday);
  for j := 1 to nstock do
    begin
        stock_shop_time[j] := plus(stock_shop_time[j-1],h);
        stock_true_time[j] := clockinv(stock_shop_time[j], opentimes,
                                  closetimes, proportions, nday);
    end;
We assume therefore that the arrays stock_true_time and stock_shop_time have been ini-
tialised.}

  for j := 0 to nstock-1 do
    begin
        p[j] := plus(stock_shop_time[j+1], -stock_shop_time[j]);
        mod_demands[j] := modmean(p[j], sigma, demand)
    end;
```

Main Routines

3. Calculating shelf capacity

{Firstly, note that if the shelf capacity is C, then the expected lost sales for the whole study period (assuming no backroom interference) is given by the function

tot_loss(C:real;sigma:real; nstock:integer; p:vec; mod_demands:vec):real.

There are three methods available.

Method 1

Use a root-finding routine to pick C so that

tot_loss(C,...) = demand*(1-CSL).

Method 2

Find which of p[0],...,p[nstock-1] is greatest (p[r], say) and then choose C so that

loss(C,sigma,p[r], mod_demands[r]) = demand*p[r]*(1-CSL).

Method 3

This is used when a retailer requires to be covered against the busiest consecutive K days trading

```
read(K);
x := 0.0;
for i := 0 to nday-1 do
begin
    y := 0.0;
    for k:= 0 to K-1 do
        begin
            l := (i+k) mod nday;
            y := y + proportions [l]
        end;
    if (y > x) then
        x := y;
end;
z := modmean (x, sigma, demand);
```

{and now we use the root-finding routine to pick C so that

```
    loss(C, sigma, x, z) = demand* x * (1-CSL). }

{By now, the program has a value of C to work with, either calculated by one of the above
methods, or input by the user. Now we can compute the expected sales in each interstock-
ing period:}
    for j := 0 to nstock-1 do
        sales[j] := demand*p[j] - loss(C, sigma, p[j], mod_demands[j]);
```

4. **Calculating target stock levels, and order-to levels**

```
    for j := 0 to ndel do
        begin
            del_true_time[j] := plus(del_true_time[j], delay_protect -epsilon);
            del_shop_time[j] := clock(del_true_time[j], opentimes, closetimes,
                                      proportions, nday)
        end;
{next, we work out the effective delivery times, and store these in del_true_time,
del_shop_time.}
    for i := 0 to MAX do
        flag[i] := -1;
    for k := 0 to ndel-1 do
    begin
        for j := 0 to nstock-1 do
        begin
            l := (j+1) mod nstock;
            if contain(del_true_time[k], stock_true_time[j], stock_true-time[l]) then
            begin
                del_true_time[k] := plus(stock_true_time[l], -epsilon);
                del_shop_time[k] := plus(stock_shop_time[l], -epsilon);
                flag[l] := k
            end;
        end;
    end;
    del_shop_time[ndel] := del_shop_time[0];
{now we get the order times into shop time}
```

```
    for k := 0 to ndel do
    order_shop_time[k] := plus(clock(order_true_time[k], opentimes, closetimes,
                               proportions, nday), -2* epsilon);

{Now the calculation of target stock levels:}
    for j := 0 to ndel-1 do
    begin
        raw_tsl[j] := 0.0;
        for i := 0 to nstock-1 do
        begin
            if  contain(stock_shop_time[i],  del_shop_time[j],  del_shop_time[j+1])
            then
            raw_tsl[j] := raw_tsl[j] + sales[i];
        end;
        target_stock_levels[j] := max(raw_tsl[j],C);
{this assignment statement must be skipped if the user has chosen to input his own
target_stock_levels}
    end;

{Now the calculation of lead-time sales:}
    for k := 0 to ndel-1 do
    begin{1}
        x := 0.0;
        for j := 0 to nstock-1 do
        begin{2}
            if  contain(stock_shop_time[j],  order_shop_time[k],  del_shop_time[k])
            then
                x := x + sales[j];
            else
                if
                contain(order_shop_time[k],stock_shop_time[j],stock_shop_time[j+1])
                then
                begin{3}
```

```
                    t := plus(order_shop_time[k], -stock_shop_time[j]);
                    z := loss(C*p[j]/t, sigma, p[j], mod_demands[j]);
                    z := t*(demand - (z/p[j]));
                    z := sales[j] -z;
                    x := x+z;
                end; {3}
            end; {2}
            lead_time_sales[k] := x;
        end; {1}
```

{It may be that the user has input target stock levels, rather than relying on the values calculated by the program. Either way, there is an array target_stock_levels containing this data.}

```
    for k := 0 to ndel-1 do
    order_to[k] := lead_time_sales[k] + target_stock_levels[k];
```

5. Lost sales

```
    {Firstly, calculate effective_tsl.}
    x := 0.0;
    k := 0;
    for j := 0 to ndel-1 do
        begin
            if (target_stock_levels[j] > x) then
                begin
                    k := j;
                    x := target_stock_levels[j]
                end;
            end;
    effective_tsl[k] := target_stock_levels[k];
    for i := 1 to ndel-1 do
    begin
        j := (k+i-1) mod ndel;
        l := (k+i) mod ndel;
        x := effective_tsl[j] -min(raw_tsl[j], effective_tsl[j]);
        effective_tsl[l] := max(x, target_stock_levels[l])
    end;
```

{From this, we calculate next the overall lost sales, and the customer service level achieved.}

```
x := 0.0;
for k := 0 to ndel-1 do
    x := min(raw_tsl[k], effective_tsl[k]) + x;
lost_sales := demand - x;
CSL_achieved := x/demand;
```

6. **Average inventory calculations**

```
for k := 0 to ndel-1 do
begin
    for j := 0 to nstock-1 do
    begin
        if (flag[j] = k) then
        begin
            backroom[j] := max(effective_tsl[k]-C, 0.0);
            shelf[j] := min(C, effective_tsl[k])
        end;
    end;
end;
```

{this has set the values of backroom and shelf at stocking times when there is a delivery. The next task is to calculate it for all stocking times, for which we must find the first stocking time which has a delivery, and work from there.}

```
i := 0;
while (flag[i] = -1) do
    i := i+1;
for j := 1 to nstock-1 do
begin
    k := (i+j-1) mod nstock;
    l := (i+j) mod nstock;
    if (flag[l] = -1) then
    begin
        x := min(shelf[k], sales[k]);
        y := shelf[k] - x;
        backroom[l] := max(backroom[k] - x, 0.0);
        shelf[l] := min(backroom[k] + y, C);
```

```
        end;
    end;
```

{The functions a and b allow us to obtain for any t in [0,1] the average amount on the shelf (respectively, in the backroom) at time t.

To calculate the average amount seen by the customers, we do a (trapezium-rule) calculation of $\int_0^1 a(t)dt$.

To calculate the average amount in the store at the end of the day's business, we take

```
for j := 0 to nday-1 do
g[j] := a(clock(j/nday)) + b(clock(j/nday));
average := 0.0;
for j := 0 to nday-1 do
average := average + g[j];
average := average/nday;
```

To find the genuine time-average, we make the integral

$$\int_0^1 (a(clock(t)) + b(clock(t)))dt. \quad \}$$

Subprogram Specifications

function clock(t: real; a: daily; b: daily; pr: daily; n: integer): real;

{given a time t in the interval (0,1), this function calculates the proportion of business done by time t. It also extends periodically to the whole line.}

```
var
      m,j : integer;
      x, sum, y : real;
begin
      m := trunc(t);
      x := t-m;
      if (x >= b[n-1]) then
            clock := 1.0
      else
      begin
            j := 0;
            sum := 0.0;
            while (x >= b[j]) do
                  begin
                        sum := sum + pr[j];
                        j := j+1
                  end;
            y := max(0.0, (x - a[j])/(b[j] - a[j]));
            clock := sum + pr[j]*min(1.0,y)
      end;
      clock := clock + m;
end;
```

We also need

function max(x: real; y: real): real

which returns the larger of x and y,

function min(x: real; y: real): real

which returns the smaller of x and y, and

function frac(x: real): real

which returns x - trunc(x), the fractional part of x.

```
function plus(x: real; y: real): real;
{this adds x to y mod1}
    begin
        plus := frac(x+y)
    end;
```

function clockinv(t: real; a: daily; b: daily; pr: daily; n: integer): real;

{this is the inverse function to clock; given t, it returns s such that clock(s) = t. The array a contains opening times, b contains closing times, and pr contains proportions of business done on the various days}

```
    var
        m,j: integer;
        x, sum: real;
    begin
        m := trunc(t);
        x := t-m;
        j := 0;
        sum := 0.0;
        while (x >= sum) do
            begin
                sum := sum + pr[j];
                j := j+1
            end;
        z := sum - x;
        clockinv := m + b[j] - (b[j] - a[j])*z/pr[j];
    end;
```

function N(z: real): real;

{this gives a good approximation to the integral from z to infinity of the standard normal distribution.}

```
var
    x,y,v: real;
begin
    y := 1/(1 + 0.23164190*abs(z));
    x := 0.3989423*exp(-0.5*z*z);
    v := 1.330274*y*y*y*y*y;
    v := v - 1.821256*y*y*y*y;
    v := v + 1.781478*y*y*y;
    v := v - 0.356538*y*y;
    v := v + 0.3193875*y;
    v := x*v;
if (z >= 0.0) then
    N := v
else
    N := 1-v
end;
```

```
function A(z: real): real
{this gives the expectation of the positive part of Y-z, where Y is a standard normal random variable}
  begin
      A := 0.3989423*exp(-0.5*z*z) - z*N(z)
  end;
```

```
function modmean(x: real; s: real; d: real): real;
```

{if x is the proportion of the business done in some interstocking interval, s is the standard deviation of the overall demand, and d is the overall mean demand, this function works out the mean of a normal random variable Y so that the expected value of Y^+ is equal to xd.}

Once again a general root-finding routing is needed. This function should return that value y such that

$$\frac{d\sqrt{x}}{s} = A\left(\frac{-y}{s\sqrt{x}}\right).$$

The value y will always be less than xd; negative values are possible.

```
function loss(C: real; sigma: real; x: real; mean: real): real;
```

```
{this calculates expected lost sales in our interstocking interval when the shelf capacity is
C and a proportion x of the business for whole study period is done}
    var
        z: real;
        begin
            z := sigma*sqrt(x);
            loss := z*A((C-mean)/z)
        end;

function tot_loss(C: real; sigma: real; nstock: integer; p:vec; mod_mean: vec): real;
    var
        j: integer;
        begin
            tot_loss = 0.0;
            for j := 0 to nstock-1 do
                tot_loss := tot_loss + loss(C, sigma, p[j], mod_mean[j])
        end;

function contain(x: real; a: real; b: real): boolean;
{this function should only be used on arguments in the interval [0,1]; it returns 'true' if x
is in the interval from a to b (with the endpoints of the interval wrapped around) other-
wise 'false'.}
    begin
        if (a < b) then
            if (a <= x) and (x <= b) then
                contain := true
            else  contain := false;
        else
            if (a <= x) or (x <= b) then
                contain := true
            else  contain := false
        nd;

function a(t: real; stock_shop_time: vec; shelf: vec; sales: vec; demand: real): real;
```

```
{this works out the average amount on the shelf at time t.}
    var
        j: integer;
    begin
        for j := 0 to nstock-1 do
        begin
        if contain(t, stock_shop_time[j], stock_shop_time[j+1]) then
            a := max(shelf[j] - demand*(t-stock_shop_time[j]), max(shelf[j]-sales[j],
            0.0))
        end;
    end;

function b(t: real; stock_shop_time: vec; backroom: vec): real;
{this works out the average amount in the backroom at time t.}
    var
        j: integer;
    begin
        for j := 0 to nstock-1 do
        begin
            if contain(t, stock_shop_time[j], stock_shop_time[j+1]) then
            b := backroom[j]
        end;
    end;
```

While the invention has been described with reference to d tails of the illustrated embodiment, these details are not intended to limit the scope of the invention as defined in the appended claims.

Claims

1. A retail shelf inventory management system comprising:

memory means for storing inventory data, said inventory data including a study period and a retail opening time and a retail closing time for each day within said study period and historical inventory data for said study period for each of a plurality of inventory items including a movement value, a customer service level, a proportion of business done for each day within said study period, and a variability of demand; and

optimizing means operatively connected to said memory means for optimizing a shelf capacity for each of said plurality of inventory items.

2. A retail shelf inventory management system as recited in claim 1 wherein said optimizing means include means for calculating an expected lost sales value for said study period and identifying said shelf capacity responsive to said calculated expected lost sales value.

3. A retail shelf inventory management system as recited in claim 1 wherein said inventory data includes a desired customer service level value to be achieved; and wherein said optimizing means include means for calculating an expected lost sales value for said study period and identifying said shelf capacity responsive to said calculated expected lost sales value and said desired customer service level value.

4. A retail shelf inventory management system as recited in claim 1 wherein said inventory data includes stocking time values and wherein said optimizing means include means for identifying an interstocking period having a maximum proportional value of business done and identifying said shelf capacity responsive to said identified interstocking period.

5. A retail shelf inventory management system as recited in claim 1 wherein said inventory data includes a desired customer service level value to be achieved and said historical inventory data includes interstocking time values and wherein said optimizing means include means for identifying an interstocking period having a maximum proportional value of business done and identifying said shelf capacity responsive to said identified interstocking period and said desired customer service level value.

6. A retail shelf inventory management system as recited in claim 1 wherein said inventory data includes a desired customer service level value to be achieved; and wherein said optimizing means include means for identifying a predetermined number of consecutive days having a maximum proportional value of business done and identifying said shelf capacity responsive to said identified consecutive days and said desired customer service level value.

7. A retail shelf inventory management system as recited in claim 1 wherein said inventory data includes a number of shelf stockings and at least a first stocking time; and further comprising means for calculating an optimum shelf stocking schedule for said study period.

8. A retail shelf inventory management system as recited in claim 1 wherein said historical inventory data includes a number of order deliveries and delivery times in said study period for each of said plurality of inventory items and further comprising means for calculating an effective delivery time and means for calculating a target stock level responsive to said calculated effective delivery time and said shelf capacity for any of said inventory items.

9. A retail shelf inventory management system as recited in claim 1 further comprising means for calculating an order to level defining an order quantity amount for any of said inventory items.

10. A retail shelf inventory management system as recited in claim 1 further comprising means for calculating an expected lost sales value responsive to a user selected shelf capacity.

11. A retail shelf inventory management system as recited in claim 1 further comprising means for calculating an expected lost sales value responsive to said optimized shelf capacity.

12. A retail shelf invent ry management system as recited in claim 1 further comprising means for calculating an average shelf inventory value responsive to said optimized shelf capacity.

13. A retail shelf inventory management system as recited in claim 1 further comprising means for calculating an average store inventory value responsive to said optimized shelf capacity.

14. A retail shelf inventory management system as recited in claim 1 further comprising means for calculating an average shelf inventory value and an average store inventory value responsive to a user selected shelf capacity.

15. A retail shelf inventory management system as recited in claim 1 further comprising input means coupled to said memory means for receiving user input selections.

16. A retail shelf inventory management system as recited in claim 1 further comprising display means coupled to said optimizing means for displaying results for a user of the system.

17. A retail shelf inventory management system comprising:

memory means for storing inventory data, said inventory data including a study period and a retail opening time and a retail closing time for each day within said study period and historical inventory data for said study period for each of a plurality of inventory items including a movement value, a customer service level, a proportion of business done for each day within said study period, and a variability of demand, a number of deliveries and order delivery times within said study period and a number of stockings and stocking times within said study period, and a customer service level to be achieved;

optimizing means operatively connected to said memory means for optimizing a shelf capacity for any of said plurality of inventory items; and

means coupled to said optimizing means for calculating an effective delivery time and an order quantity amount for any of said inventory items responsive to said optimized shelf capacity.

18. A retail shelf inventory management system as recited in claim 17 wherein said optimizing means include means for calculating an expected lost sales value for said study period and identifying said shelf capacity responsive to said calculated expected lost sales value.

19. A retail shelf inventory management system as recited in claim 17 wherein said inventory data includes a desired customer service level value to be achieved; and wherein said optimizing means include means for calculating an expected lost sales value for said study period and identifying said shelf capacity responsive to said calculated expected lost sales value and said desired customer service level value.

20. A retail shelf inventory management system as recited in claim 17 wherein said inventory data includes a desired customer service level value to be achieved and said historical inventory data includes interstocking time values and wherein said optimizing means include means for identifying an interstocking period having a maximum proportional value of business done and identifying said shelf capacity responsive to said identified interstocking period and said desired customer service level value.

FIG. 1

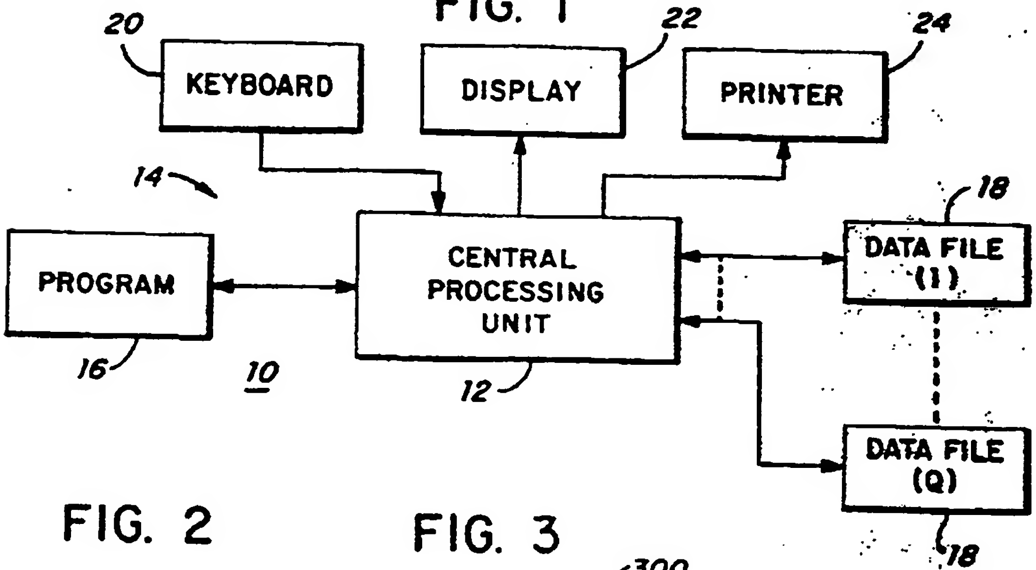


FIG. 2

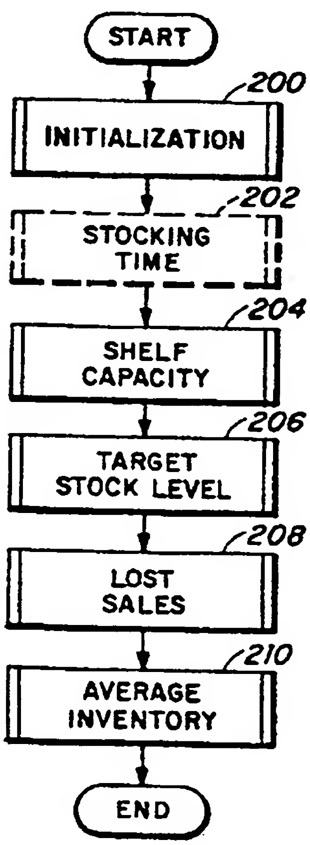


FIG. 3

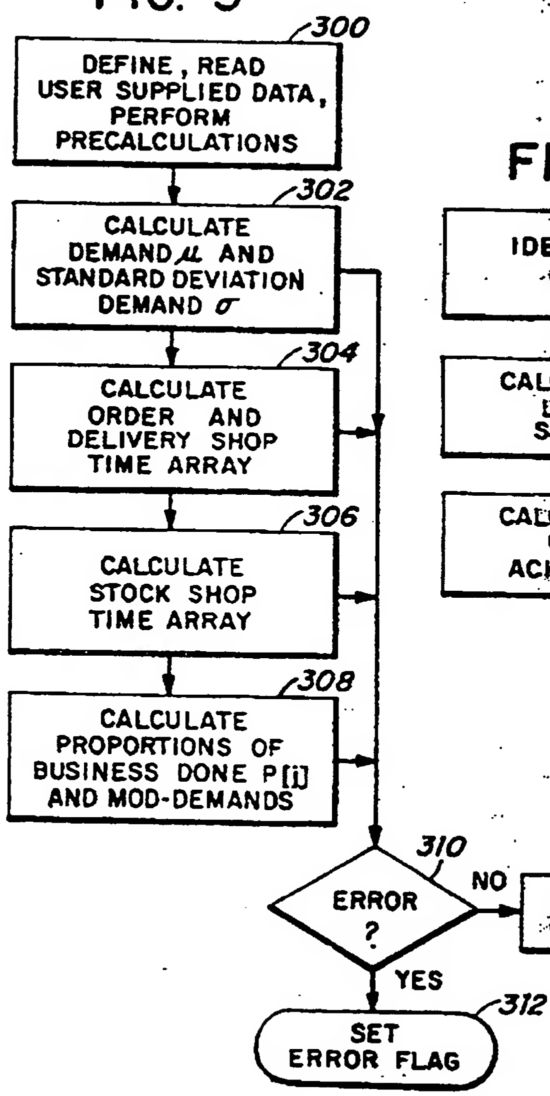


FIG. 6

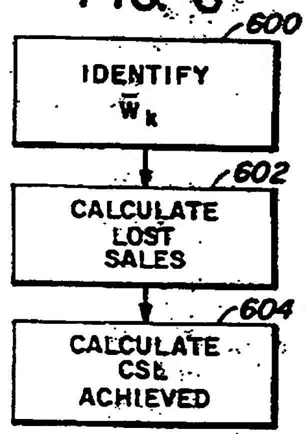


FIG. 4

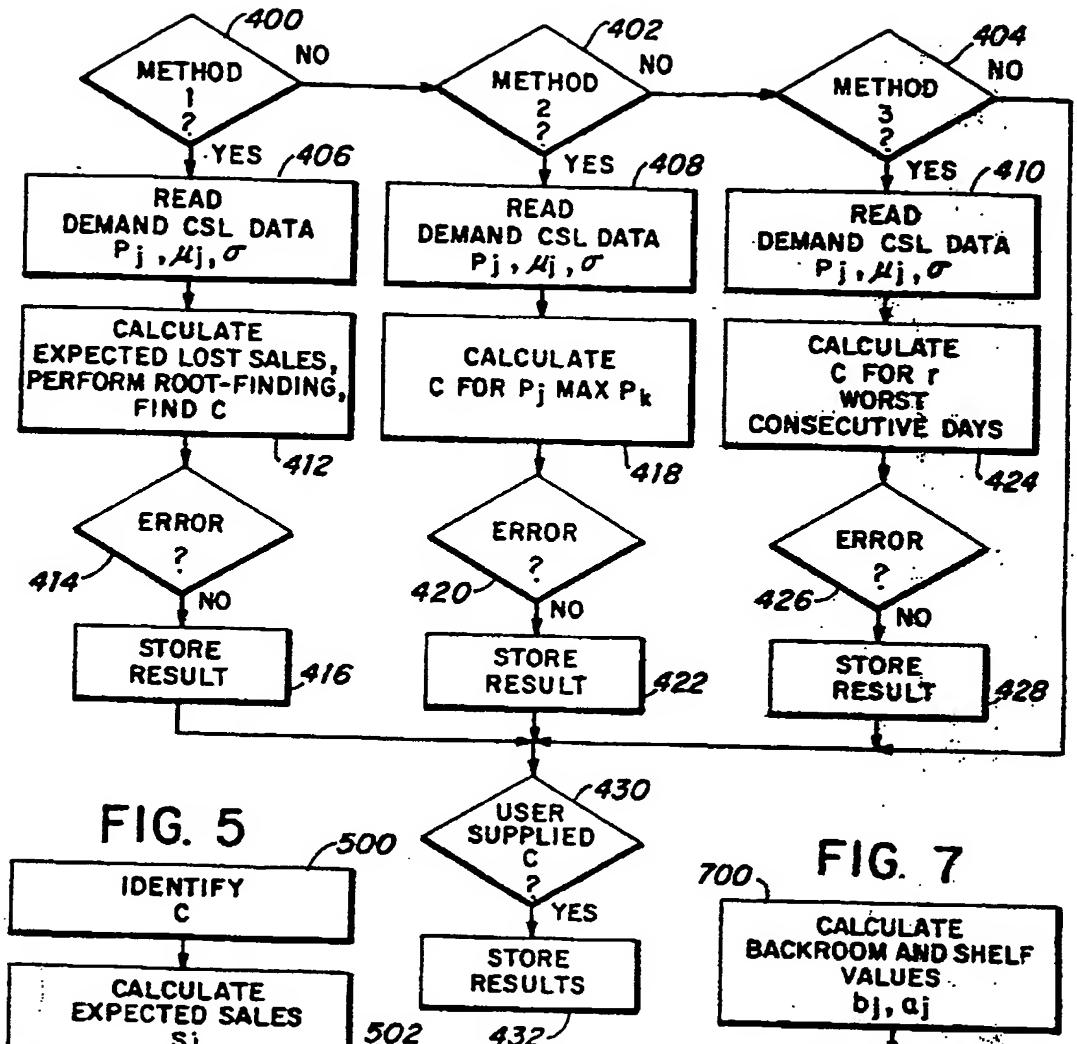


FIG. 5

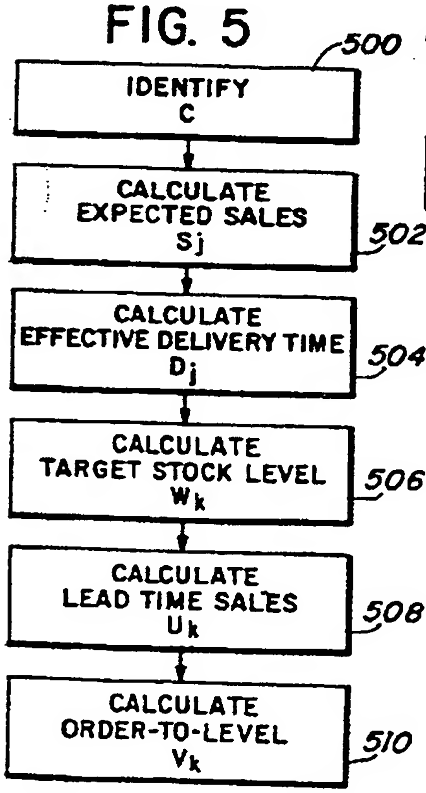
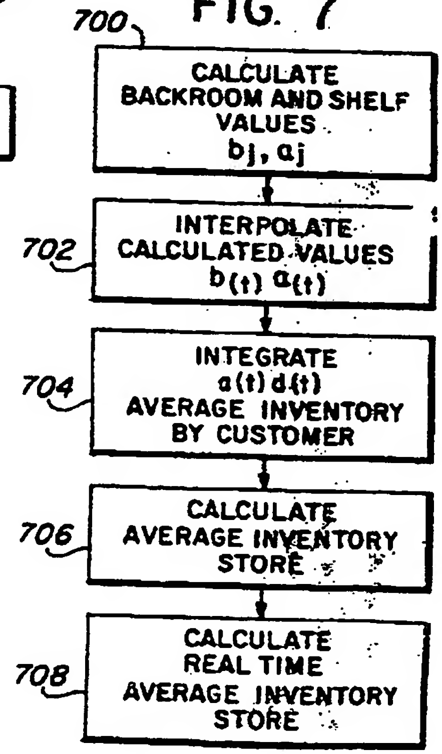


FIG. 7



INTERNATIONAL SEARCH REPORT

International Application No. PCT/US90/00652

I. CLASSIFICATION OF SUBJECT MATTER (if several classification symbols apply, indicate all) [6]

According to International Patent Classification (IPC) or to both National Classification and IPC

IPC (5): G06F 15/24

U.S. CL: 364/403; 235/385

II. FIELDS SEARCHED

Minimum Documentation Searched [7]

Classification System	Classification Symbols
U.S.	364/403; 235/385

Documentation Searched other than Minimum Documentation to the Extent that such Documents are Included in the Fields Searched [8]

III. DOCUMENTS CONSIDERED TO BE RELEVANT [9]

Category *	Citation of Document, [11] with indication, where appropriate, of the relevant passages [12]	Relevant to Claim No. [13]
A	Management Science, Volume 33, no.6, issued June 1987, F.J. Arcelus et al. 'Inventory policies under various optimizing criteria and variable markup rates', see pages 756-62.	1-20
A	Proceeding, American Institute of Industrial Engineers, 1985 Annual International Engineering Conference, Los Angeles, CA, I.W. Kabak et al. 'Utilization of inventory and satisfaction of demand' see pages 516-521.	1-20

* Special categories of cited documents: [10]

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosu... ...e, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

IV. CERTIFICATION

Date of the Actual Completion of the International Search	Date of Mailing of this International Search Report
12 MARCH 1990	10 MAY 1990
International Searching Authority	**Signature of Authorized Officer**
ISA/US	CLARK JABLON for